

# Non-Perturbative Renormalisation using Domain Wall Fermions

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The viability of the Non-Perturbative Renormalisation (NPR) method of the Rome/Southampton group is studied, for the first time, in the context of domain wall fermions. The procedure is used to extract the renormalisation coefficients of the various quark bilinears, as well as the four-fermion operators relevant to the  $\Delta S = 2$  effective Hamiltonian. The renormalisation of the  $\Delta S = 1$  Hamiltonian is also discussed.

## 1. INTRODUCTION

Domain Wall Fermions [2](DWF) are an attractive approach to Lattice QCD that allows exact vector and axial symmetry at finite lattice spacing, the latter in the limit of an infinite fifth dimension. This property greatly simplifies the renormalisation properties of lattice operators. However, realistic simulations are performed with a fifth dimension of finite extent (16 sites for the results presented below) and so potential breaking of chiral symmetry must be considered [3].

Below a brief overview of a preliminary study of the renormalisation properties of several operators using the regularisation independent technique of the Rome/Southampton group (RI/MOM) [1] is given.

## 2. QUARK PROPAGATOR

The RI/MOM scheme requires the calculation of the operators of interest between external quark states, in a fixed gauge, at high momenta. For many quantities of interest this requires only knowledge of the momentum space quark propagator from a fixed origin,  $S(p)$ , and so simulations are relatively “inexpensive”.

\*This manuscript has been authored under contract number DE-AC02-98CH10886 with the U.S. Department of Energy. Work done in collaboration with T. Blum, P. Chen, N. Christ, M. Creutz, C. Dawson, G. Fleming, A. Kaehler, T. Klassen, C. Malureanu, R. Mawhinney, S. Ohta, S. Sasaki, G. Siegert, C. Sui, A. Soni, M. Wingate, P. Vranas, L. Wu, and Yu. Zhestkov.

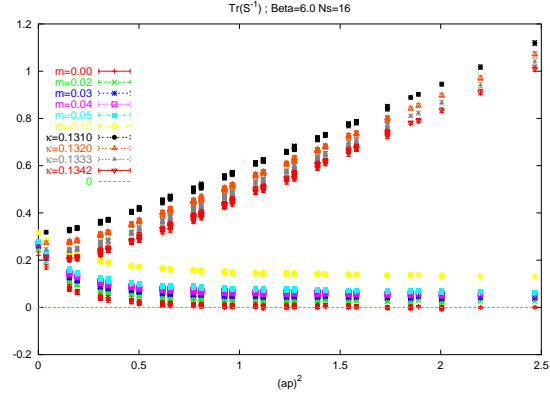


Figure 1.  $Z_q M_{RI}$

From just the propagator itself the quark renormalisation factor,  $Z_q^{1/2}$ , and, more interestingly, the quark mass may be calculated. An RI/MOM scheme mass may be defined from the inverse propagator as

$$M_{RI} = \frac{1}{Z_q} \frac{Tr S^{-1}(p^2)}{12} = Z_m m. \quad (1)$$

This definition of the mass is extremely sensitive to the presence of explicit breaking of chiral symmetry at  $O(a)$ , in which case an additive renormalisation is needed [6,12]. Fig. 1 contrasts  $Z_q M_{RI}$  for both non-perturbatively improved Wilson fermions <sup>2</sup> and DWF, showing explicitly the good chiral symmetry of the DWF, with no additive term apparent within errors. Both lattices had dimensions  $16^3 \times 32$  with  $\beta = 6.0$  and

<sup>2</sup>Thanks to A.P.E. for this data

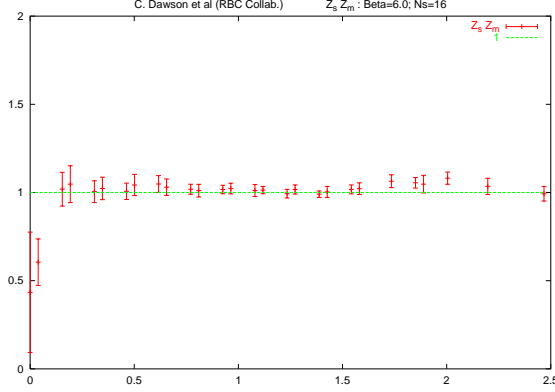


Figure 2.  $Z_S \times Z_M$

the results shown are for 37 and 52 gauge configurations respectively.

### 3. FERMION BILINEARS

The renormalisation factors for the flavour non-singlet fermion bilinears in the RI scheme may be calculated from [10]

$$\frac{Z_\Gamma}{12 Z_q} \text{Tr} [\Gamma \langle q | \bar{\psi} \Gamma \psi | \bar{q} \rangle_{AMP}] |_{p^2=\mu^2} = 1, \quad (2)$$

with  $\Gamma = \{S, V, P, A, T\}$ .

Values for these quantities and comparison to perturbation theory will appear in a forthcoming paper. For the moment only properties relevant for the extraction of the quark masses will be mentioned. In particular, if both the vector and axial symmetries are respected, the relation  $Z_S = 1/Z_m$ , should hold. Fig. 2 shows  $Z_S \times Z_m$  vs  $(ap)^2$ . The fact that this relation holds so well both confirms the predictions of perturbation theory [14] and gives confidence in the method used to extract the strange quark mass in [7], where more details may be found.

### 4. $\Delta S = 2$ HAMILTONIAN

The parity conserving part of the  $\Delta S = 2$  Hamiltonian is proportional to

$$O_{VV+AA} = \bar{s} \gamma_\mu \gamma_5 d \bar{s} \gamma_\mu \gamma_5 d + \bar{s} \gamma_\mu d \bar{s} \gamma_\mu d. \quad (3)$$

In the continuum this renormalises multiplicatively, but in the presence of explicit chiral symmetry breaking four other operators may mix;  $O_{VV-AA}$ ,  $O_{SS-PP}$ ,  $O_{SS+PP}$  and  $O_{TT}$  [11].

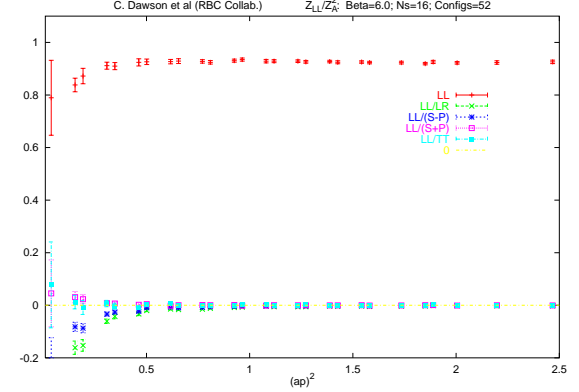


Figure 3.  $Z_{\Delta S=2}/Z_A^2$

An RI-scheme renormalisation condition may be imposed on this set of operators by constructing the matrix elements of  $O_i$  between external quark states,

$$\Lambda_{\alpha\beta\gamma\delta}^i = \langle 0 | O_i \psi_\alpha \bar{\psi}_\beta \psi_\gamma \bar{\psi}_\delta | 0 \rangle_{amp}, \quad (4)$$

taking the projection of these matrix elements with a complete parity conserving basis in spinor space,

$$P_{\Gamma \otimes \Gamma} [\Lambda] = \Gamma_{\alpha\beta}^i \Gamma_{\gamma\delta}^i \Lambda_{\alpha\beta\gamma\delta}^j \quad (5)$$

and then requiring that, for renormalised operators, this be equal to its free case value,

$$Z_{ij} P_{(\Gamma \otimes \Gamma)_j} [\Lambda^k] = F_{ik}. \quad (6)$$

Fig. 3 shows the overall renormalisation and mixings for  $O_{VV+AA}$ , and as can be seen, in a simulation with a fifth dimension of 16 points there is essentially no mixing from chirally disallowed operators. This may be contrasted to the case for Wilson fermions in which mixings are of order 10% [11]. It should be noted, that even with a fifth dimension of only four sites, while the mixing with chirally disallowed operators becomes appreciable, the magnitude of the mixing coefficients is smaller than that for Wilson fermions.

### 5. $\Delta S = 1$ HAMILTONIAN

The renormalisation of the parity conserving part of the  $\Delta S = 1$  Hamiltonian is a much more challenging problem on the lattice than that of the fermion bilinears or the  $\Delta S = 2$  Hamiltonian,

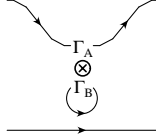


Figure 4. “eye” diagram

but also of much greater interest phenomenologically [8]. The difficulty in evaluating the renormalisation conditions for this Hamiltonian can be attributed to the appearance of “eye” diagrams such as that shown in Fig. 4. These diagram introduce two problems.

Firstly they cannot be constructed from merely the momentum space quark propagator from a single origin, as they require a completely disconnected quark propagator transformed into momentum space,

$$S(p, q) = \sum_{x, y} e^{-ip \cdot x} S(x, y) e^{iq \cdot y}. \quad (7)$$

This may be calculated by inverting the Dirac equation using a fixed momentum source, but this “costs” a matrix inversion for every momenta,  $q$ , needed. For this reason only four fixed momenta have been used during this exploratory study.

Secondly such diagrams allow mixing with lower dimensional operators, including, but not limited to, off-shell “Equation of Motion” operators [5], such as;

$$\bar{s}(-\overleftrightarrow{D} + m)d + \bar{s}(\overleftrightarrow{D} + m)d. \quad (8)$$

All such operators must be subtracted (or shown to be irrelevant) before the equivalent of Eq.(5)<sup>3</sup> may be applied. This may be done in principle by, for example, studying the matrix elements of the effective Hamiltonian between external  $s$  and  $d$  quark states [9], but in practice this procedure only seems possible in the case of a lattice action that respects chiral symmetry due to the proliferation of possible operator mixings otherwise.

Fig. 2 shows three sample renormalisation factors, on a  $16^3 \times 32 \times 16$  lattice, using 288 gauge configurations, for  $m = 0.04$ . The subtraction of the lower dimensional operators has currently not been completed, but preliminary results suggest that the effects of these terms are small.

<sup>3</sup>A complete basis of projectors in both spin and flavour space must be used in this case.

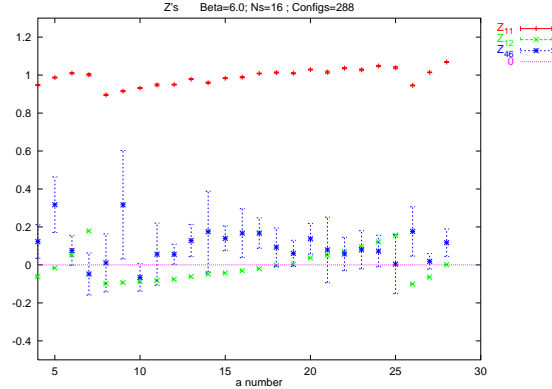


Figure 5. Non-subtracted renormalisation factors

## 6. CONCLUSIONS

A preliminary study of the RI/MOM scheme applied to DWF has performed. The results seem encouraging, especially in the (low) level of chiral symmetry breaking exhibited.

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